RETTEVEJLEDNING

Written Exam, Summer school, Economics summer 2011

Micro 3

Final Exam

July 25th, 2011

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Problem 1

Consider the following normal form game:

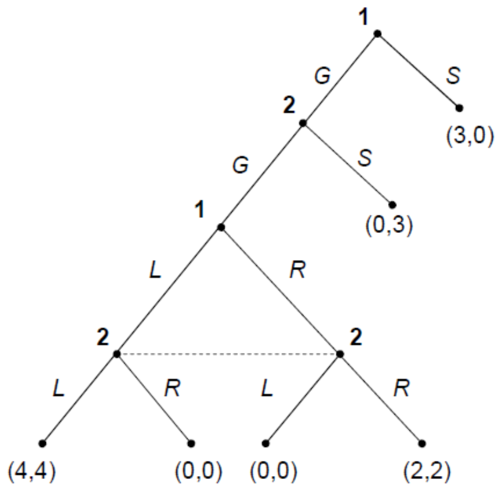
	A_2	B_2	C_2	D_2
A_1	4,3	5,0	3,1	2,0
B_1	1,1	2,5	1,4	4,0
C_1	2,2	4,3	5,5	6,1

(a) Analyze the game by iterated elimination of strictly dominated strategies. Describe each step. Which strategies survive?

D_2 is dominated, e.g., by C_2. B_1 is dominated by C_1. When B_1 is removed, then B_2 is dominated by C_2. Surviving strategies are A_1 and C_1 for player 1, and A_2 and C_2 for player 2.

Problem 2

Consider the game given by the following game tree:



(a) How many subgames are there in the game (excluding the game itself)?

Two – one beginning after player 1 has moved *G*, and one beginning after both player 1 and 2 has moved *G*.

(b) Describe the set of possible strategies sets for each player.

Each player should choose between *G* and *S* and between *L* and *R*, thus there are four different strategies $\{`GL, GR, SL, SR\}$. The strategies starting with S are outcome equivalent, and just providing "plans-of-action" $\{GL, GR, S\}$ is not a severe mistake, but makes answering (d) difficult.

(c) Write down the normal-form ("matrix form") of the game and find all pure strategy Nash equilibria.

	GL	*GR*	*SL*	*SR*
GL	4,4	0,0	0,3	0,3
GR	0,0	2,2	0,3	0,3
SL	3,0	3,0	3,0	3,0
SR	3,0	3,0	3,0	3,0

Nash equilibria (by inspection in the matrix):
$(GL, GL), (SL, GR), (SL, SL), (SL, SR), (SR, GR), (SR, SL), (SR, SR)$

(d) Find all the pure strategy subgame-perfect Nash equilibira (SPE) of the game.

$(GL, GL), (SR, SR)$

Problem 3

In a duopoly each of the two firms produces a homogenous good for which the inverse demand is given by:

$$p = 7 - Q + A_1 + A_2,$$

where Q is the total quantity sold, p is the price and $A_i \geq 0$ is firm i's advertising effort, $i = 1,2$. The cost of making an advertising effort of A_i is $(A_i)^2$. The production of firm i, $q_i \geq 0$, is produced at zero costs, and $Q = q_1 + q_2$. The game between the firms has two stages. First each firm chooses its advertising effort *without* knowing the effort of the other firm. When both firms have chosen their efforts, the total effort is revealed. Then each firm chooses its produced quantity *without* knowing the other firm's production. Then a market clearing price is formed, and each firm receives a profit, which is the firm's payoff in the game.

(a) Sketch an extensive form of the game. Find the payoffs π_1 and π_2 to the firms as functions of A_1, A_2, q_1 and q_2.

$\pi_i(A_1, A_2, q_1, q_2) = (7 - q_1 - q_2 + A_1 + A_2)q_i - A_i^2$

A sketch should indicate the timing of decisions and the knowledge that players have when deciding. In particular it should be clear that *A*s are chosen first and then *q*s, and that *A*s are known when choosing *q*s.

(b) Find for given advertising efforts A_1 and A_2 the Nash equilibrium in the subgame starting when firms have choosen A_1 and A_2. Find, as function of A_1 and A_2, the quantities q_1 and q_2 produced, and the market price.

$$q_1(A_1, A_2) = q_2(A_1, A_2) = \frac{7 + A_1 + A_2}{3}$$

$$p(A_1, A_2) = \frac{7 + A_1 + A_2}{3}$$

(c) Find the subgame perfect equilibrium of the entire game. State the market price, and the production, advertising effort, and profit of each firm in this equilibrium.

$p = 3$, $q_1 = q_2 = 3$, $A_1 = A_2 = 1$, $\pi_1 = \pi_2 = 9 - 1 = 8$

Problem 4

Consider the following game in strategic form, where a is some real number.

	L	R
U	5 , 1	4 , a
D	2 , 1	5 , 0

(a) Find for all values of a all (pure and mixed) Nash-equilibria of the game.

For $a<1$, (U,L) is the unique Nash equilibrium

For $a>1$, the game has a unique Nash equilibrium in which player 1 plays U with probability $\frac{1}{a}$, and D with probability $1-\frac{1}{a}=\frac{a-1}{a}$, and player 2 plays L with probability 0.25, and R with probability 0.75.

For $a=1$, (U,L) is a Nash equilibrium, and also strategy combinations where player 1 plays U and player 2 plays L with probability at least 0.25 are Nash equilibria.

Assume that player 2 knows the value of a, where as Player 1 just knows that a = 3 with probability ½ and = –3 with probability ½.

(b) Explain that this situation can be modelled as a Bayesian game, i.e., describe the players, actions, types, prior and utility functions in the Bayesian game modeling this situation.

Players 1 and 2

Actions $A_1=\{U,D\}$, $A_2=\{L,R\}$

Player 1 just has one type $T_1=\{t_1\}$, player 2 has two types $T_2=\{t_2^H,t_2^L\}$, where t_2^H means that a = 3.

Prior $p_1(t_2^H|t_1)=p_1(t_2^L|t_1)=0.5$.

Utilities are as given in the matrix with a = 3 for t_2^H and a = –3 for t_2^L.

(c) Find the Bayesian Nash equilibrium of this Bayesian game.

$s_1^*(t_1)=U, s_2^*(t_2^H)=R, s_2^*(t_2^L)=L$

Problem 5

Three firms, A, B and C, use a similar technology, even though they produce different products and are not competing in the product market. They negotiate to pool some of their research and development resources (laboratories and engineers) in a development project that can improve the production efficiency in each of the firms. They expect that after having covered the development costs, they can increase their joint profit by 100 from undertaking the project.

Each firm can also attempt the project alone or with just one of the other firms, but that is expected to be relatively more costly. In particular firm A expect that it can earn a profit 40 by doing the project alone, where as firms B and C being somewhat smaller and lacking a core facility that firm A holds, can each only earn 20 from undertaking the project alone, and 50 in total if they work together. If A undertakes the project with either B or C, the joint profit of the pair is expected to be 70.

(a) Think of this situation as a cooperative game and write down the value of all coalitions.

$v(\{A\}) = 40, v(\{B\}) = v(\{C\}) = 20$

$v(\{A, B\}) = v(\{A, C\}) = 70, v(\{B, C\}) = 50$

$v(\{A, B, C\}) = 100$

(b) Find the core of this game.

The convex hull of $\{(50,20,30),(50,30,20),(40,30,30)\}$. Providing it in a figure is fine.

(c) Discuss briefly how the core is relevant for predicting the outcome of the negotiations between the firms.

If a proposed distribution of the surplus does not lie inside the core, then one or two of the firms can obtain a higher payoff by making a smaller joint venture or going for the project alone – thus such a proposal is unlikely to be agreed upon in the grand coalition.